Schiaparelli (M. E. G. V.). Considerazioni sul Moto Rotatorio del Pianeta Venere. 8vo. *Milano* 1890. With Four other Excerpts in 8vo. The Author.

Two Folio Volumes containing MS. Correspondence on Terrestrial Magnetism, between the Rev. Humphry Lloyd and Sir E. Sabine and others. 1833—1878. Mrs. Lloyd.

"The Rupture of Steel by Longitudinal Stress." By CHARLES A. CARUS-WILSON. Communicated by Professor G. H. DARWIN, F.R.S. Received March 10,—Read March 27, 1890.

[PLATES 2, 3.]

In a paper read before the Royal Society on June 16, 1881, Professor G. H. Darwin stated: "It is difficult to conceive any mode in which an elastic solid can rupture except by shearing, and hence it appears that the greatest shearing stress is a proper measure of the tendency to break" ('Phil. Trans.,' 1882, p. 99).

In this paper, I have recorded the results of some experiments made with a view to throwing light on the question raised by Professor Darwin.

The experiments were conducted in the mechanical laboratory at the Royal Indian Engineering College, Cooper's Hill, the machine used being a hundred-ton single lever hydraulic testing machine by Messrs. Buckton & Co., of Leeds. This machine is fully described in Unwin's 'Testing of Materials of Construction,' p. 133.

All tension experiments were performed on circular specimens fitted with a screw thread at each end, on to which were screwed steel nuts resting in spherical seatings to ensure directness of pull.

The shearing experiments were conducted on specimens screwed throughout their entire length into three steel blocks, and tested in double shear, *i.e.*, the two outside blocks were pulled in the opposite direction to the inside block—perpendicularly to the axis of the specimen, and the bar sheared in two places at once.

The idea of rupture necessarily implies an overcoming of resistance. If the power of steel to resist rupture under tension were a constant quantity, the conception of rupture would be simple, for we should only have to increase the stress up to the required amount, and the bar would break; but the resistance to rupture appears to be a function of the amount of flow that has taken place (see experiments described in Table II). If we take two axes, OX, OY, to represent respectively the elongation per unit of length, and the stress per unit of area of transverse section of elongated bar, we obtain the

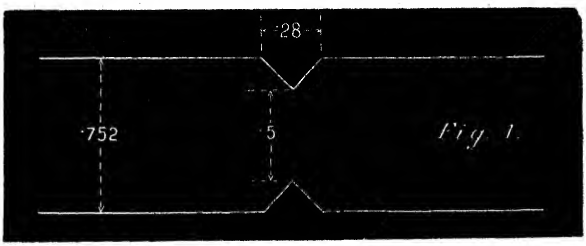


Fig. 1.

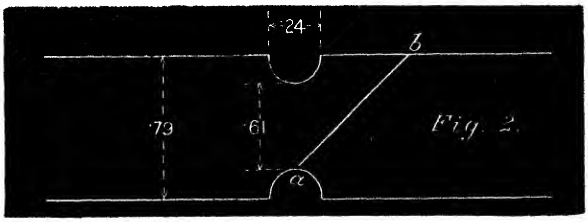


Fig. 2.

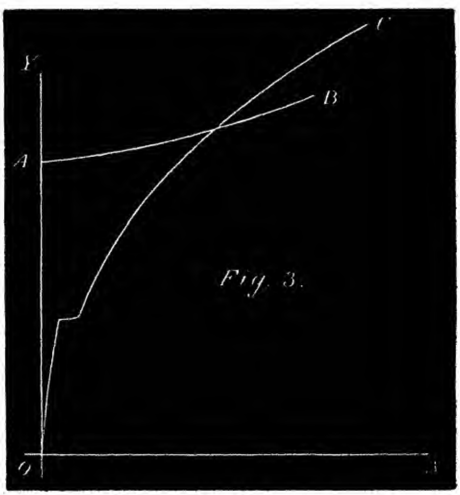


Fig. 3.

true stress strain curve OC for the steel of which the bar is composed (fig. 3). We may refer to the same axes the curve AB showing the increase in the resistance to rupture with elongation per unit of length—and when AB cuts the curve OC the bar must break, *i.e.*, when the stress per unit of area of transverse section becomes greater than the breaking stress corresponding to that degree of elongation. If the two curves do not meet, the bar will continue to draw out—as is the case with lead.

The method that is ordinarily adopted for estimating the tensile

strength of a metal, viz., by dividing the maximum load by the original area, is a purely conventional method, and does not represent any real stress whatever; it simply shows what load would be sustained by a given section before it broke, and though it is no doubt a useful figure for engineers to know, it does not tell us anything about the actual stress at fracture; this can only be arrived at by dividing the load on the specimen at the point of rupture by the contracted area measured after the specimen has broken. In this paper, the breaking stress will always be measured in this way, and will be referred to as the "true tensile strength" of the metal. This is what M. Considère in his 'L'Emploi du Fer et de l'Acier' (Paris, Dunod, 1885) calls "résistance de striction."

It is well known that when a bar is subjected to tension—the stress not being uniformly distributed, as, for instance, when the pull is not central—the mean stress borne by the bar at rupture is less than it would be if the stress had been uniformly distributed.

It is stated in Thomson and Tait's 'Natural Philosophy,' Part II, p. 258, that "a solid of any elastic substance, isotropic or æolotropic . . . . . experiences infinite stress and strain in the neighbourhood of a re-entrant edge or angle, when influenced by any distribution of force, exclusive of surface tractions infinitely near the angles or edges in question."

Three steel bars, numbered 829, 830, and 831, were taken and cut in three pieces; one piece was tested plain, and the second piece with a V-groove turned on it (see fig. 1). The tool cutting the V-groove was made with its cutting edges at about 90°, and the point as fine as possible. The results are given in Table I. It is clear that the V-groove is very prejudicial. For the same reason a V-groove with a rounded angle must be prejudicial, though not to such an extent, since the distribution of stress is more uniform. Specimen No. 834 was cut in three pieces and tested, one plain and one with a groove of the same shape as Nos. 829, 830, 831, but with the point of the cutting tool just rounded off. The strength of the grooved bar is now 0·95 of the plain. Similarly, with specimens 822 and 50, the grooved pieces have strengths of 84 and 89 respectively. These experiments show that the mean stress at rupture diminishes as the angle of the groove becomes more acute.

When, however, we come to test specimens with a groove as in fig. 2, we find that they are stronger than the plain specimens.

Table I gives the results of such experiments on seven steel bars (including the four already mentioned). Each bar was cut in three pieces.

The plain bar was tested first, and the groove then cut in the second piece to the same diameter as the contracted area in the plain piece, so as to secure as much as possible similarity of conditions.

Table I.

Laboratory No.	O.	V.	U.	O_1.	V_1.	U_1.	c.
829	73·5	53·1	83·2	100	72	113	53·2
830	73·4	57·8	87·0	100	79	119	50·5
831	70·4	58·0	84·6	100	82	120	50·0
834	59·5	56·7§	63·0	100	95	106	52·4
822	70·0	58·8§	77·5	100	84	111	52·5
439	60·5	—	74·5	100	—	123	36·4
50	58·1	53·7§	69·4	100	89	119	61·4

O, V, U are the tensile strengths of the plain, V-grooved, and U-grooved bars respectively in tons per square inch. O_1, V_1, U_1 the same, taking that of O as 100. c is the percentage contraction of area of the plain bar.

Fig. 1 shows the dimensions of the V-groove adopted in all cases, except in Nos. 822, 834, 50 §, where the corner was just rounded off by the cutting tool.

Fig. 2 shows dimension of the U-groove.

It will be seen that in every case the U-grooved specimen is stronger than the plain, the average superiority being 16 per cent.

The effect of the U-groove by itself in producing non-uniformity of stress—as in the V-groove—would tend to make the U-grooved bar break at a lower stress than the plain bar, where the stress must be very nearly uniformly distributed, but, in spite of this prejudicial action, the U-grooved bar is the stronger.

This phenomenon is quite distinct from that mentioned by many writers, who have pointed out that a grooved specimen is stronger than a plain specimen of the same material—the stresses being reckoned in the conventional manner, viz., maximum load divided by original area (*cf.* Unwin, 'Testing of Materials of Construction,' p. 82, and Burr, 'Elasticity and Resistance of Materials of Engineering,' p. 230).

The reason of this is that the so-called tensile strength depends on the amount of drawing out before local contraction begins, and since in a plain bar the general contraction of area may be considerable, the actual load on the specimen, at the maximum, is much smaller than on a bar of the same metal in which the drawing out is suppressed —owing to the groove.

In the experiments quoted above, I have discounted altogether the contraction of area, and considered only the actual stress on the section at rupture.

It is possible that in some cases the metal at the groove is stronger

than at any other point in the bar; this would tell in its favour against a plain bar which is free to break at the weakest spot. The steel on which these experiments were conducted was very homogeneous, and the variation of strength within a short distance on the same bar would not be more than 4 or 5 per cent., so this would scarcely account for the phenomena.

On the other hand, a grooved specimen is at a disadvantage when compared with a plain specimen for four reasons :—

(i) The stress is much more unevenly distributed over the least section in the grooved bar than in the plain.

(ii) If the pull is not exactly parallel to the axis of the bar, bending stresses are induced, which are very prejudicial in the grooved bar, whereas their effect is largely neutralised in the plain bar by the ready flow of the metal.*

(iii) The load at rupture can be observed with accuracy in testing the grooved bar, for it breaks off short, and the required load is also the maximum load. In testing the plain bar, however, in consequence of the very rapid contraction of area immediately before rupture, the load has to be reduced, in order to keep the lever horizontal; sometimes the load cannot be run back quick enough, and the bar may break while the lever is resting on the bottom stop, so that too high a load may be observed as the load of rupture; this would tend to give a higher breaking stress in the plain bar than was actually the case.*

(iv) The grooved bar has a crystalline fracture. The plain bar has a silky fracture. Experiments will be quoted later on to show that the ultimate resistance to rupture is less, the more crystalline is the steel at the moment of rupture.

Careful measurements have been made of the test piece No. 831 (the others being very similar), to ascertain the least area of all planes passing through any point in the narrowest section at 45° to the axis (i) in the grooved bar, (ii) in the plain bar; the section of these planes is shown at *ab* in fig. 4; the diameter at the narrowest section was the same in both specimens. The ratio of the area of this plane in the grooved bar to that in the plain was found to be 183 : 100.

If, now, rupture is an overcoming of a resistance to shearing, the grooved bar ought to be stronger than the plain in the ratio of 183 to 100, other things being the same in both bars.

For the resistance to shearing, at rupture, will be the resistance of all the planes similar to those shown in the figures, equally inclined to the axis, and if the area of any one of these planes in the grooved

* This is only stated as a possible source of error; no result was accepted if there was any suspicion of its being thus influenced.

FIG. 4.

bar is 183/100 of any one in the plain, the resistances of all the planes will be in the same ratio.*

In this specimen (No. 831), the ratio of the true breaking stresses is actually 120 : 100. But there are four causes tending to reduce the strength of the grooved specimen, as has been shown above, viz.:—The non-uniformity of stress, the possibility of a pull not perfectly longitudinal, the danger of observing too high a load on the plain specimen, and, lastly, the crystalline nature of the steel at fracture in the grooved specimen.

* This assumes that the shearing stress is uniform over both of these oblique planes; the probability of this supposition is discussed in a paper on "The Distribution of Flow in a Strained Elastic Solid," published in the 'Philosophical Magazine,' for June, 1890.

All these causes tend to reduce the superiority of the grooved bar; the effect of the non-uniformity of stress is, no doubt, the greatest; in fact, as has been shown, if the groove is of V-shape, this causes the plain bar to be the strongest, so that it is quite conceivable that the non-uniformity of stress in the U-grooved bar, with other causes mentioned, might reduce the strength from 180 : 100 to 120 : 100 of the plain bar.

The following are the experiments referred to, in order to prove that the ultimate resistance of steel to shearing diminishes inversely with the drawing out:—

Six pieces of the same steel bar were taken and drawn out under tension to varying percentages of their length; they were then prepared as shearing specimens, as explained above, and tested in double shear, with the following results:—

Table II.

Bar. No.	Extension per cent.	Cont. of area per cent.	Area of section sheared.	Shearing stress at rupture.
			sq. in.	tons per sq. in.
1	0·0	0·0	0·317	20·50
2	5·0	4·8	0·322	20·10
3	10·0	9·1	,,	20·21
4	15·0	13·0	,,	21·77
5	20·0	16·6	,,	22·12
6	20·2	16·7	,,	22·37

Thus the ultimate resistance to shearing increases with the drawing out. Taking the contraction of area as a measure of the flow, No. 831 specimen contracted 50 per cent. on the plain bar and 28·1 per cent. on the U-grooved bar, so that the former would be at an advantage, compared with the latter, in this respect.

It appears, then, that the load that can be borne by a given section of steel without rupture can be increased by thickening the bar above and below the section in question, though, if the angle of the groove be too acute, the reverse is the effect. But the prejudicial action of a groove, owing to non-uniformity of stress produced, is the same in a U- or V-groove, and differs only in amount, so that the U-groove, by itself, could not strengthen the bar, but must weaken it; yet in spite of this, the U-grooved bar is stronger than the plain bar. The increase of strength, then, must be due to the added material; but in no way could such added material strengthen such a section, and enable it to stand a greater load, if rupture is produced by a certain intensity of tensile stress; we cannot diminish the mean stress on the section by thickening the bar above and below; on the

contrary, we increase the stress over part of the section, and it is the maximum stress that we have to reckon with, for the bar will begin breaking there and tear across.

There is no doubt that the added material increases the resistance to shearing, and I am, therefore, led to the conclusion that it is this increased resistance to shearing that causes the increase in strength; in other words, by adding material above and below the section, the shearing stress for elements lying in the section is certainly reduced, and, at the same time, the strength is certainly increased, the conclusion drawn being that the true measure of the tendency to break is the greatest shearing stress.

The fact that longitudinal tension is equivalent to a uniform dilating tension and a shearing stress, and that by the means above described we can diminish the latter, without altering the former, and thereby strengthen the bar, are strong reasons for supposing that Professor Darwin's statement is correct, and that it is the shearing stress produced by longitudinal stress that causes rupture.

If it be true that the rupture of a steel bar under tension is determined by the greatest shearing stress, we should expect to find that a definite relation existed between the ultimate resistance to direct shearing and the same to direct tension.

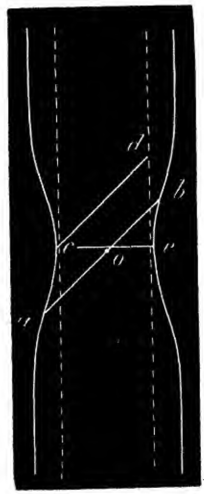
Much has been written about the relation of these two resistances, but the conclusions drawn are very misleading, since the tensile strength considered has been that calculated in the conventional manner, which has, as has been shown, no real significance and is no real stress.

By a well known theorem, the greatest shearing stress is equal to one-half the longitudinal stress; we should then expect to find that one-half the true tensile stress at rupture was equal to the stress at rupture in a shearing experiment on a piece of the same steel.

If the steel be soft, it will contract locally before breaking; hence the greatest shearing stress will be less than half the longitudinal stress in the ratio of the sections of a cylindrical and contracted bar, cut by planes parallel to *dc* and *ab* (fig. 5) respectively, the two bars having the same cross section at *coe*; in other words, in the ratio of $\sqrt{2}$ (area across *coe*) to (area across *aob*), where $boe = 45°$. If this ratio be called θ, and the true tensile stress at rupture be p, the shearing stress at rupture in a shearing experiment should be equal to $\frac{1}{2}p\theta$.

Table III gives the resuls of some experiments made to investigate this question. The tensile and shearing experiments were made respectively on pieces of steel cut from the same bar; the former were made on circular specimens screwed at each end, and resting on nuts bearing on spherical seatings; the shearing specimens were screwed

FIG. 5.



along their entire lengths, and tested in double shear in screwed steel blocks to eliminate bending.

The 3rd column gives ω_1, the original section of the tension specimen; the 4th gives c, the percentage contraction of area in the

TABLE III.

Laboratory No.	Original Dimensions.	ω_1.	c.	$\frac{1}{2}p$.	θ.	$\frac{1}{2}p\theta$.	ω_2.	s.
801	$1\frac{1}{2}'' \times 1\frac{3}{8}''$	0·542	43·7	32·5	0·88	28·6	0·305	28·7
900	,,	0·628	46·3	32·8	0·87	28·5	1·021	30·1
42	1″ round	0·400	61·7	26·2	0·81	21·2	0·322	22·7 }
43	,,	0·389	64·0	27·9	0·79	22·0	0·322	22·4 }
970*a*	,,	0·381	54·1	37·1	0·81	30·0	0·322	31·2 }
970*b*	,,	0·386	53·4	36·3	0·83	30·1	0·322	31·0 }
898*a*	,,	0·384	53·1	37·9	0·83	31·5	0·312	31·8 }
898*b*	,,	0·384	50·5	37·6	0·83	31·2	0·312	31·2 }
198	,,	0·348	33·9	31·2	0·93	29·0	0·322	30·6
199	,,	0·348	50·5	34·2	0·85	29·3	0·322	28·6
49	$1\frac{1}{4}''$ round	0·640	56·5	22·1	0·82	18·1	1·021	18·3

[*Note.*—Those bracketed together are from the same bar. Nos. 198 and 199 each give the mean of two experiments on the same bar. Nos. 801, 900, 970*a* and *b*, 898*a* and *b*, 198, and 199 are from steel bars prepared by the Patent Nut and Bolt Company, the steel being made by the Barrow Steel Company. Nos. 42 and 43 are from bars of soft crucible steel manufactured by Messrs. Osborn at the Clyde Steel Works, Sheffield. No. 49 is from a bar of Lowmoor iron.]

tension experiment; the 5th gives half the true tensile stress; the 6th gives the values of θ, deduced by measurement in each case; the 7th gives $\frac{1}{2}p\theta$; the 8th gives ω_2, the area of the specimen in the shearing experiment; and the 9th column gives s, the intensity of shearing stress at rupture in the shearing experiment.

It will be seen that in every case $\frac{1}{2}p\theta$ is very nearly equal to s, *i.e.* the shearing stress at rupture in a tensile experiment is very nearly equal to the ultimate resistance to shearing in a pure shearing experiment.

There are, however, two points to be considered before accepting the result of these experiments.

The distribution of stress over the section of rupture in the tension experiment has been assumed constant, whereas it is not actually so. I find, by actual measurement, that the area of a plane section at 45° to the axis passing through the centre of the narrowed section bears to the area of a parallel plane passing through a point on the circumference, the ratio of 100 to 108, in a bar which has contracted 50 per cent., so that the shearing stress is rather greater at the centre, and hence the value of $\frac{1}{2}p$, given above, is too small by about 4 per cent.

On the other hand, it has been pointed out to me by Professor Darwin that the distribution of stress in the shearing experiment is probably not uniform, being greater in the neighbourhood of the application of the stress.

Experiments were made with two pieces of Lowmoor iron, cut off the same bar, and prepared as shearing specimens in the ordinary way, and tested in double shear, one with an area to be sheared of twice 1·039 square inch, and the other of twice 0·322 square inch. The result was as follows: Large section, shearing stress at rupture—(i) 18·7, (ii) 18·9; mean, 18·8 tons per square inch. Small section, stress at rupture—(i) 20·1, (ii) 20·6; mean 20·35. Giving the latter as 8·2 per cent. stronger than the former. The smaller the section the more uniform will be the stress, and with the small section employed in the experiments quoted in Table III the stress is probably nearly uniform.

It would seem, then, that the possible errors due to the unequal distribution of stress in the tensile and shearing experiments would nearly balance one another, and that we may regard these results as tending to confirm the theory that the greatest shearing stress is the proper measure of the tendency to break.*

* I have made experiments of a similar kind on cast iron. Great care was taken in casting to secure uniformity, by casting the bars upright and cutting off the spongy top; they were cast with two heads which were turned to fit spherical seatings. The shearing specimens were cut off bars from the same cast. The bars in tension were 10 inches long between the shoulders, and turned throughout their length.

It will now be necessary to enquire how far the appearance of the fracture of a steel bar affords evidence of its having broken by shearing.

In a bar of circular section and uniform thickness throughout its length, every plane at 45° to the axis opposes an equal resistance to the tangential stress caused by direct tension. Hence, there is no one plane or planes along which the bar would be more ready to break by shearing than along any other plane, provided that the material was of uniform strength throughout. If, however, the bar be gradually thinned at a certain point, this will no longer be the case; it has been shown on p. 252, that the area of a plane at 45° to the axis passing through the centre of the narrowed section is less than the area of a plane passing through any other point in that section; hence, there will be a surface formed by a complete cone of 45°, with apex at the centre of the narrowed section, which will oppose a less resistance to rupture by shearing than any other similar cone with apex at any other point. This cone is shown in section in fig. 6 at *gof—aob*. We should then expect to find rupture result in a fracture formed by a cone and crater, or, since there is nothing to determine along which part of the cone rupture will take place, we may expect to find the cone irregularly broken up, part on one end, and part on the other.

This narrowing of girth at one point always accompanies the rupture of soft steel, and we invariably find such a cone and crater; figs. 5 and 6, Plate 2, and 6 and 7, Plate 3, are good examples.

[*Note.*—The rupture of cast iron in compression by shearing is of course well known. Fig. 2, Plate 2, shows the cone of shearing very well.]

In flat bars of soft steel, this shearing action is still more marked. Here the surface of the least resistance is a plane at 45° to the axis and making 90° with the thin side of the bar; it is evident that in a bar whose width is considerable compared with its thickness, and which has suffered considerable local contraction, this plane has the least area of all planes at 45° to the axis passing through any point in

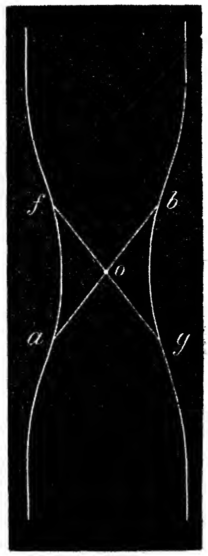
	Sectional area.	Breaking stress. Tons per sq. inch.	Mean ditto.
Tension....	1·047	10·40	10·40
	0·980	10·40	
Shearing ...	0·317	6·14	5·46
	0·327	4·78	

The ratio of the former to the latter breaking stress being 1·9.

The mean crushing stress was 41·5 tons per square inch; diameter of specimen, 0·875 inch; length, 1·5 inch.

In the 'Proceedings of the Institution of Civil Engineers,' vol. 90, p. 406, Messrs. Platt and Hayward give results of shearing and tensile tests of cast iron, from which it appears that the ratio of the breaking stresses is 2·2.

FIG. 6.



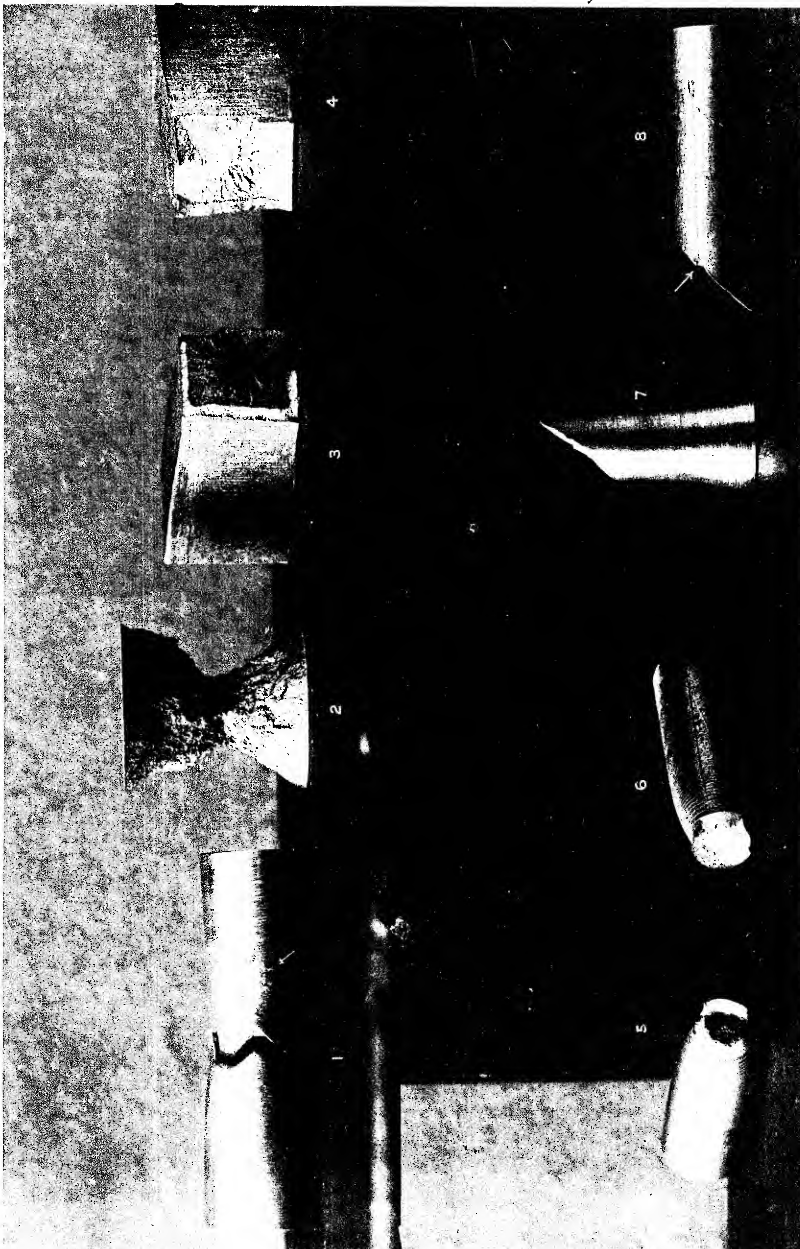
the contracted section. The result is, that in flat bars of soft steel the fracture is almost invariably as shown in figs. 4 and 5, Plate 3. Fig. 8, Plate 3, shows a flat bar of soft steel just about to rupture by shearing along a plane $\perp$ the width of the bar —resulting no doubt from an accidental weakness in that direction.

In the cases considered above, the steel has been of sufficiently uniform quality to allow of the fracture taking place over a surface of least resistance to shearing; but, unless this condition be fulfilled, the form of the fracture will be quite different.

Figs. 7 and 8, Plate 2, show the fracture of a brass bar where the plane of least resistance to shearing has been determined by a punch mark (opposite the arrow) on the surface. Fig. 1, Plate 2, shows a steel bar where the apex of the cone is at the circumference, owing to the presence of a weak spot there.

Every fracture is caused by the presence of a more or less well defined weak spot; the stress is greatest at this spot, and the material tends to tear in a plane $\perp$ the axis passing through this spot. This tearing action can be observed by drilling a small hole in a steel plate, and straining it. The plate pinches in near the hole, and gives way first on each side of the hole, and then tears right across. The experiment may be stopped before the tear has reached the sides.

When the steel is hard, this tearing continues in the plane in



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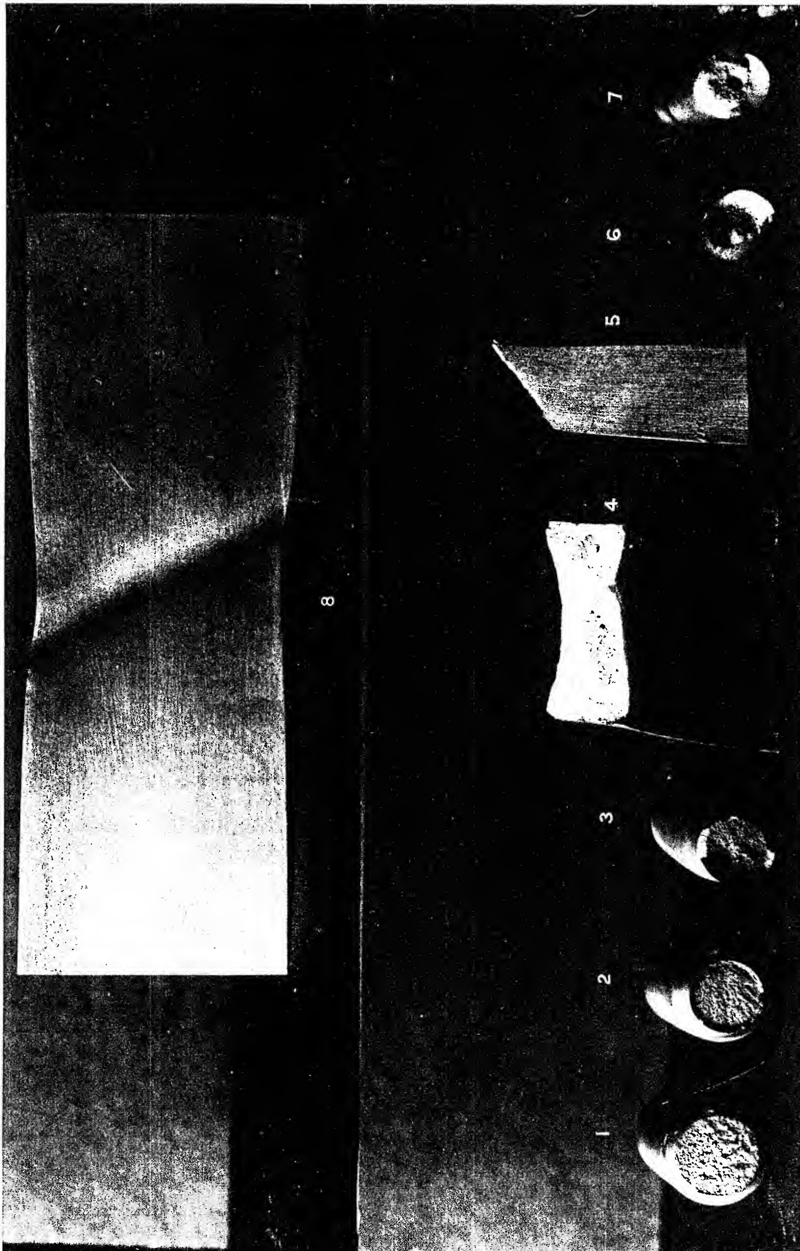

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which it commenced, *i.e.*, perpendicular to the axis; but when the steel is soft, the plane of the tear gradually tilts over and coincides with the surface of least resistance to shearing, *i.e.*, becomes inclined at 45° to the axis.

Now, at rupture, an originally soft bar is harder in the centre of the narrowed section than at the circumference, where the drawing out has been less; hence, fracture commences at the centre perpendicular to the axis, and tears outwards until it reaches the softer material, when it will continue along a surface of least resistance to shearing, *i.e.*, along a surface formed by the intersection of two cones. Hence, we find the fracture of a soft steel bar consisting of a crater with a more or less extended base; see figs. 5 and 6, Plate 2, and 6 and 7, Plate 3.

The harder the steel, at the outset, the broader will be the base of the crater, until, in very hard steels, there is only a rim or crown left round the edge; and in the hardest steels all trace of the surface of least resistance to shearing disappears.

[*Note.*—I have employed the term "hard" in the sense usually understood, *i.e.*, where the "hardness" is measured by the value of the limit of elastic resistance.]

"Photometric Observations of the Sun and Sky." By WILLIAM BRENNAND. Communicated by C. B. CLARKE, F.R.S. Received October 30,—Read December 11, 1890.

1. In the publications of the Society from 1859 to 1870, many communications by Sir Henry Roscoe on this subject will be found. Of these, the most important bearing directly on my observations are—

a. Bunsen and Roscoe, "On the Direct Measurement of the Chemical Action of Sunlight," in 'Phil. Trans.,' 1863, pp. 139–160.

It is proved, *inter alia*, that equal shades are produced in photographically sensitised paper by equal products of intensity of light × time of insolation. The preparation of a photographic paper which shall always possess the same degree of sensitiveness is carefully described.

b. Roscoe, "On a Method of Meteorological Registration of the Chemical Action of Total Daylight," in 'Phil. Trans.,' 1865, pp. 605–631 [Bakerian Lecture].

The law is stated, *inter alia*, that light of intensity 50 acting for 1 second has the same effect as light of intensity 1 acting for 50 seconds.

The mechanical arrangement for exposing the paper horizontal, or by the aid of a vertical drum, is explained.


Fig. 1


Fig. 2


Fig. 3

Fig. 4.

FIG. 5.

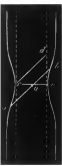
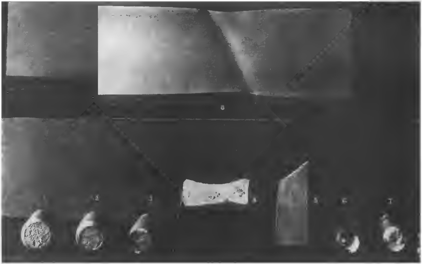


Fig. 6.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